A New(?) Relationship Between Wind Speeds and Return Periods

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Introduction

The model described below is certainly not entirely new and is heavily based on previous work of NJ Cook and RI Harris on analysis of extreme winds. It has also been reported in part in a paper by MD Burton (Arup) and AC Allsop (then Arup) presented to the regional America's 11th Wind Engineering conference in 2009.

The discussion below considers only one parent mechanism causing an omni-directional wind but is flexible enough to be developed for multi-mechanism and variable direction cases. The model also remains applicable for any wind climate where strong winds are dominated by one wind mechanism.

The model takes account of knowledge of the occurrence rate of wind-storms to precondition the windspeed v. return period relationship, which may be difficult to estimate reliably from extreme value analysis of limited sets of measured data. A major benefit is that the number of events can be calibrated using regional data which is likely to be less affected by local statistical variability than a single anemometer record.

Parent Wind Model

The parent distribution of strong winds from a particular windstorm mechanism can in many cases be described using a Weibull distribution with an exponent of two. It can be difficult coming to this conclusion from looking at unsorted wind data, but it has long been observed that fitted values from common large-scale cyclonic storm winds(1) have values which are quite close to this value.

The assumed cumulative probability distribution(2) is thus given by:

$$P(v) = 1 - e^{-\left(\frac{v}{c}\right)^2} \qquad (1)$$

Distribution of Maxima of Multiple Storms, N_{st}

Given a parent distribution, the distribution of maximum values from a number of storms may be derived by raising the cumulative distribution to the power of the number of storms, N_{st}. i.e.

$$\left(P(v)\right)^{N_{st}} = \left(1 - e^{-\left(\frac{v}{c}\right)^2}\right)^{N_{st}} \qquad (2)$$

It has been fairly standardly stated in past wind engineering courses that the distribution of speed converges on a Type-I Gumbel distribution. Less well understood is that the convergence is both slow and poor. JD Holmes in particular has long pointed out that good quality data with a single parent distribution shows a rising curve of diminishing slope when

[1] Other kinds of wind do not always fit this model, especially where trade- or diurnal-winds add to effects of storms carried with them. But it seems to work for tropical- and well as extra-tropical cyclones.

[2] Note the Weibull 'c' velocity associated with the peak winds during the passage of a storm is higher than the 'c' value of the entire parent distribution. This is because 'a storm' is associated with varying windspeeds over its duration. This might be a good topic for a future paper.

plotted on a horizontal log scale, rather than the expected straight line, although this is easily obscured when looking at mixed climate data or just due to single extreme events which often do not seems to fit the rest of the data. (i.e. Statistically it is not likely that the worst extreme will fit well.)

But RI Harris demonstrated the known mathematical principle that exponential probability distributions converge on Type-I Gumbel distributions the most rapidly.

This can be achieved most easily by replacing P(v) by P(v^2) in the equation (2) above.

$$\left(P(v^2)\right)^{N_{st}} = \left(1 - e^{-\left(\frac{v}{c}\right)^2}\right)^{N_{st}} \tag{3}$$

Velocity-squared extremes have consequently been used by NJ Cook for assessing extreme winds in the UK since the mid-1980s.

Alternative Fits

A similar form of curved-fit may be achieved using more complex 3-parameter Frechet and GEV EV methods. However, it is well known the second parameter (slope) can be difficult to fit when confronted by real data. For practical purposes the third (curvature) parameter needs to be determined by other methods.

But the v-squared fit gives a reasonable and rational level of curvature with no need to fit an extra parameter. A further advantage of the v-squared model is that the slope may be estimated from the storm-occurrence rate and this allows regional data to be used for this purpose.

Simplification for Practical Use

The model is valid only for stronger winds, hence the value of 'c' in the equations cannot be reliably measured directly. The value of N_{st} on the other hand can be estimated by looking only at numbers of events which represent the observed storm passage. While there is some judgment involved in this, it turns out that results are not particularly sensitive to the exact value, but rather to the logarithm of this number.

A further assumption is that we are interested in the return period value of the wind speed, which can be taken as the mode of the EV model. i.e.

$$P(y = 0) \text{ of } e^{-e^{-y}} = 1/e \tag{4}$$

Equating the equations (3) and (4) and re-arranging we can get

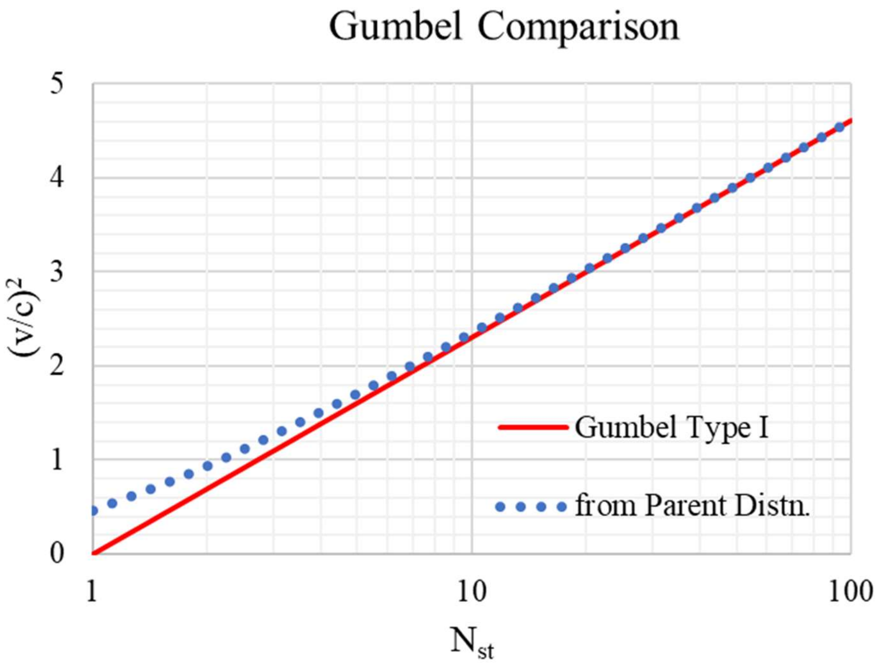
$$-L_n\left(1 - e^{-\frac{1}{N_{st}}}\right) = \left(\frac{v}{c}\right)^2 \tag{5}$$

It can be shown that this formula is closely equivalent to Gumbel Type-I as N_{st} increases.

e.g. For large N_{st}, $e^{-\frac{1}{N_{st}}}$ converges on $(1 - 1/N_{st})$ resulting in the expected form of

$$L_n N_{st} = (v/c)^2 \tag{6}$$

Substituting N_{st} = 10, $L_n(10) = 2.302..$. compares with 2.352.. using the left side of equation (5) above. Also see the figure below.



Using the formula for large N_{st}, 'c' can be eliminated if the wind speed corresponding to a particular return period, R, e.g. one year or fifty years, is known, together with a reasonable estimate of the number of storms per annum, N_a.

e.g.
$$\frac{V_Y}{V_R} = \sqrt{\frac{L_n(N_a Y)}{L_n(N_a R)}} \tag{7}$$

This may be reduced to the form of the BS6399-2 and EN1991-1-4 formulas if it is recognised that $L_n(N_a)$ for extra-tropical cyclones, the main strong wind storm type in northern Europe, is approximately '5', corresponding to about 1 event every 3 days on average.

i.e. The formula above may be rearranged as

$$\frac{V_Y}{V_R} = \sqrt{\frac{L_n N_a + L_n Y}{L_n N_a + L_n R}} \tag{8}$$

If $L_n N_a$=5 then this is further rearrangeable to the EN/BSI form (due to Cook) by dividing by 5 top and bottom inside the square-root as

$$\frac{V_Y}{V_R} = \sqrt{\frac{1+0.2 L_n Y}{1+0.2 L_n R}} \tag{9}$$

Other types of extreme wind, e.g. tropical-cyclones, may be associated with rather smaller N_a values, sometimes less than one per year. For low value of N_{st} the full formula should be used.

Conclusion

These ideas may be developed for multiple types of storm and to include effects of wind directionality. (Both N_a and storm strength vary with direction and individual storms typically cause winds from a range of directions. This is also important when considering effects of orography.)

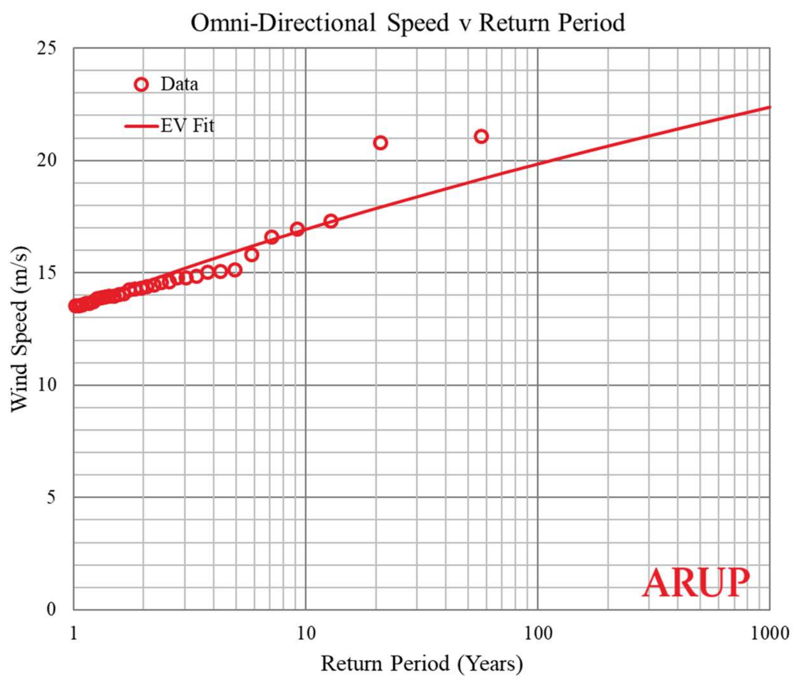
It is important from the above that the expected slope (increase of wind strength with return period) may be estimated using knowledge of the rate of storm occurrence and that such estimations will often be more accurate than estimation from single anemometer sources due to natural variation of extremes. Regional knowledge can and should also be used.

This relationship with N_a is valid using the assumption of a v-squared exponential distribution of extreme winds, but it does not depend on fitting the full parent distribution.

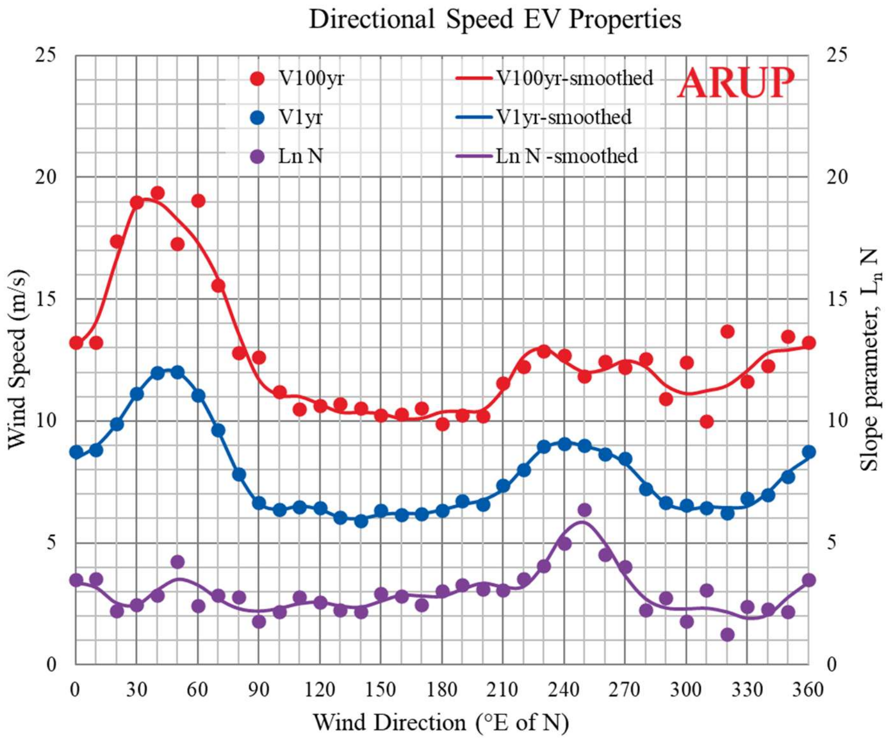
Any more complex formulation based on a linear-velocity EV function will inevitably be more difficult to calibrate due to the lack of theory for the slope and curvature parameters.

> 'Occam's razor' (raison, *fr* = logic?) says (and I paraphrase in modern English), 'When confronted by similarly valid theories, choose the simplest.' The reason is that this allows the best opportunity for ease of use and further development.
>
> PS 'Occam's razor' was not intended as an invitation to use poor solutions because they are easier, but clearly this last temptation will always exist, as will the temptation to overcomplicate, which he is resisting – all part of a normal scientific progress.

An example case



This has a slope parameter of L_n N = 3.9 (~50 events/annum) compared with EN code recommended 5.   (Note The extreme storm in this case also caused rare damage of a known much longer recurrence interval than the duration of the recordings, which was 30 years.)



Note the stability of the predicted V_{1yr} speeds compared to the V_{50yr} predictions, the improvement through smoothing the slope between directions, and the residual variability between wind directions.

NB The data was from an Italian site subject to Tramonta/Foehn winds which were clearly directional. (No exposure correction was made.)